Example 1 A customer visiting the suit department of a certain store will purchase a suit \( S \) with probability 0.22, a shirt \( S H \) with probability 0.3 and a tie \( T \) with probability 0.28. The customer will purchase both a suit and a shirt with probability 0.11, both a suit and a tie with probability 0.14 and both a shirt and a tie with probability 0.1. A customer will purchase all 3 items with probability 0.06. A customer will purchase all 3 items with probability 0.06. What is the probability that a customer will purchase a) none b) exactly 1.

Example 2 What is the probability of two people having the same birthday? What is the probability that no two of \( n \) people have the same birthday?

Section 2.4

A discrete sample space is a sample space which consists of a finite or a countably infinite set of possible outcomes. In such a case we can list the distinct elements of \( S \)

\[
S = \begin{cases} 
\{s_1, s_2, s_3, \ldots \} & \text{if } S \text{ is countably infinite} \\
\{s_1, \ldots, s_n\} & \text{if } S \text{ is finite with } n \text{ elements}
\end{cases}
\]

Let \( p_i = P(\text{outcome is } s_i) = p(\{s_i\}) = p(s_i) \)

A discrete probability function is any function \( P \) such that

1. \( 0 \leq P(s) \forall s \in S \), i.e. \( 0 \leq P(s_i) \) for all \( i \in \{1, 2, 3, \ldots\} \) if \( S \) is countably infinite, or for all \( i \in \{1, \ldots, n\} \) if \( S \) is finite with \( n \) elements.

2. \( \sum_{s \in S} P(s) = 1 \), i.e.

   (a) For a countably infinite space \( 1 = \sum_{i=1}^{\infty} P(\{s_i\}) = \sum_{i=1}^{\infty} p_i \), and

   (b) For a finite sample space with \( n \) elements, \( 1 = \sum_{i=1}^{n} P(\{s_i\}) = \sum_{i=1}^{n} p_i \).

Let event \( A \) = a subset of distinct elements of \( S \), then

\[
P(A) = \sum_{\forall s \in A} P(s).
\]

Math review

Geometric series: \( 1, x, x^2, \ldots, \)

\[
\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}.
\]

Integration: substitution

\[
\int f(g(x))dg(x) = F(g(x))
\]

where \( \int f(x)dx = F(x) \) and \( dg(x) = g'(x)dx \).

Integration by parts

\[
\int f(x)dg(x) = f(x)g(x) - \int g(x)df(x).
\]