# Math 421 Midterm 1 Fall 2002

Name:________________________

1. (36 points) Let $z = \dfrac{10}{\sqrt{3} - i}$. Compute the following (in cartesian or polar form):

   a) The polar form of $z$.

   b) $|z^3|$

   c) $\mathrm{Log}(z^9)$

   d) All values of $z^{\frac{1}{3}}$.

   e) All values of $z^{2i}$.

2. (18 points) a) Find the image, under the principal branch of $\mathrm{Log}(z)$, of the set

$$\{z \text{ such that } |z| = 2 \text{ and } z \neq -2\}$$

   (circle of radius 2, with the point $-2$ removed).

   b) Find the image of the vertical line $x = 2$ under the function $f(z) = e^{iz}$.

3. (18 points) a) Compute $\cos(i)$.

   b) Find all solutions of the equation $\cos(z) = 10$.

4. (18 points) a) Prove that the function

$$u(x, y) \;=\; x^3 - 3xy^2 - 2x + e^{-y}\cos(x)$$

   is harmonic on the whole of $\mathbb{R}^2$.

   b) Find a harmonic conjugate $v$ of the function $u$.

   c) Find an entire function $f(z)$ such that $Re(f) = u$. Your answer must be expressed as a function of $z = x + iy$, not $x$ and $y$.

5. (10 points) Let $f(z)$ be an entire function, whose real and imaginary parts satisfy the following relation

$$\mathrm{Re}(f) = 2\mathrm{Im}(f).$$

   Prove that $f$ must be a constant function. *Hint: Use the Cauchy-Riemann equations to prove that $f'(z) = 0$.*