**Math 132 Final Exam**
**Spring 2003**

Your name __________________________

ID number __________________________

Your section __________________________

**Note:**

- No notes, no books.
- It is **not sufficient** to simply write down the answers. You must **explain how** you arrive at your answers.
- You have **TWO HOURS.**

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}, \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

| | GRADE | |
|---|---|---|
| #1 | (a) | (b) |
| | (c) | |
| #2 | (a) | (b) |
| | (c) | |
| #3 | | |
| #4 | (a) | (b) |
| | (c) | |
| #5 | (a) | (b) |
| #6 | | |
| TOTAL | | |

#1. Compute the following integrals $\boxed{\textbf{algebraically}}$. SHOW YOUR WORK!

(a) [5 points] $\displaystyle\int_0^{\pi/4} \sin^3\theta\cos^3\theta d\theta$

(b) [5 points] $\displaystyle\int_0^{1} \sqrt{2-x^2}dx$

(c) [5 points] $\int te^{2t}dt$

#2. Determine whether each of the following is $\boxed{\textbf{convergent}}$ or $\boxed{\textbf{divergent}}$. EXPLAIN YOUR REASONING.

(a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n + 1}$

(b) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+2)(n+3)}$

(c) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{1/n}}$

#3. [10 points] Find $\boxed{\textbf{the first three terms}}$ of the Taylor series for the function $f(x) = x\ln(x^2+1)$ with $\boxed{\textbf{center } \boldsymbol{a = 1}}$.

#4. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{(n+1)!}$.

[5 points] (a) Determine the radius of convergence of this power series.

[5 points] (b) Show that this power series converges for $x = 1/3$.

[5 points] (c) How many terms do you need to take to estimate the infinite series in Part (b) above to within 0.001? SHOW YOUR WORK!

#5. [5 points] Sketch the region $\boxed{\textbf{in the first quadrant}}$ that lies $\boxed{\textbf{inside}}$ the polar graph $r = \cos 2\theta$ and $\boxed{\textbf{outside}}$ the polar graph $r = 1/2$. Determine all points of intersections.

[5 points] Determine the area of this region.

#6. [10 points] Find the volume of the solid obtained by rotating the curve $y = \ln x$ for $1 \leq x \leq 2$ about the $x$-axis. (Hint: use integration by parts)