Workshop on
"Geometry and topology of smooth 4-manifolds"

Max Planck Institute for Mathematics, Bonn, June 3–7, 2013

SCHEDULE

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9:30-10:30	Némethi	Furuta	Hambleton	Piergallini	Li
11:00-12:00	Teleman	Mrowka	Chen	Saeki	Ruberman
13:45-14:45	Owens	Manolescu	JUNIOR	Gay	Plamenevskya**
15:00-16:00	Sunukjian	Manolescu	TALKS*	Gompf	

(*) On Wednesday afternoon there will be short talks by finishing/recent PhDs.
(**) Last talk on Friday will be between 12:15-13:15 instead.

Abstracts

- **A. Némethi: Lattice cohomology and normal surface singularities**

  We review the definition of the lattice cohomology of links of normal surface singularities, and its connection with other topological invariants (Seiberg-Witten invariants, Heegaard-Floer homology, equivariant Ehrhart polynomials) and some analytic invariant (sheaf cohomology of natural line bundles).

- **A. Teleman: Determinant line bundles in non-Kählerian geometry and instanton moduli spaces over class VII surfaces**

  We use recent results of Jean-Michel Bismut on the curvature of the determinant line bundle in non-Khlerian geometry to study a non-Kählerian version of the Fourier-Mukai transform. We apply these ideas and gauge theoretical techniques to describe certain instanton moduli spaces on minimal class VII surfaces. We show how our instanton moduli spaces can be used to prove existence of curves on these surfaces.

- **B. Owens: Immersed disks and knot invariants**

  The four-dimensional clasp number of a knot K in the three-sphere is the minimum number of double points of an immersed disk in the four-ball bounded by K. We obtain bounds on this and on two related invariants: the slicing number and the concordance unknotting number. The main tool we use is Heegaard Floer homology, and in particular a slightly strengthened version of Ozsváth-Szabó's unknotting number one obstruction. This is joint work with Saso Strle.

- **N. Sunukjian: Surfaces in 4-manifolds: Concordance, Isotopy, and Surgery**

  The smoothly embedded surfaces in a 4-manifold play a central role in understanding the smooth structures on that manifold. There are two clues that this might be true. First, any given class in the second homology can be represented by a smooth embedded surface, but the minimal genus of such a representation depends on the smooth structure. Second, every smooth structure can be constructed via certain kinds of surgeries on surfaces. In this talk, we hope to approach the question of surgery on surfaces through the eyes of concordance. Restricting our attention to the case of simply connected 4-manifolds, we'll look at the following 3 questions. 1. When are two surfaces concordant? (A: When they represent the same homology class.) 2. When are two surfaces (topologically) isotopic? (A: Often when the complement has cyclic fundamental group.) 3. When is surgery on a given surface diffeomorphic to surgery on another surface? (A: When they are 0-concordant.)

- **M. Furuta: Seiberg-Witten Floer K cohomology and a 10/8-type inequality for spin 4-manifolds with boundaries**

  Recently Manolescu formulated Pin(2)-equivariant Seiberg-Witten Z/2-cohomolgy for spin rational homology 3-spheres. We give a K-theory version of Manolescu's construction and, as an application, show a 10/8-type inequality for a compact spin 4-manifold whose boundary is the disjoint union of spin rational homology 3-spheres. This is joint work with Tian-Jun Li.

- **T. Mrowka: Webs, Foams and Instantons**

- **C. Manolescu: The triangulation conjecture (part I)**

  The triangulation conjecture stated that any n-dimensional topological manifold has a homeomorphism to a simplicial complex. It is true in dimensions at most 3, but false in dimension 4 by the work of Casson and Freedman. In this first talk I will sketch the proof that it is also false in higher dimensions. This is based on previous work of Galewski-Stern and Matumoto, who reduced the problem to a question in low dimensions (the existence of elements of order 2 and Rokhlin invariant one in the 3-dimensional homology cobordism group). The low-dimensional question can be answered using a variant of Floer homology.

- **C. Manolescu: The triangulation conjecture (part 2)**

  In this second talk I will give more details about the disproof of the conjecture. I will describe the construction of Pin(2)-equivariant Seiberg-Witten Floer homology for rational homology 3-spheres equipped with a spin structure. This is based on

finite dimensional approximation and Conley index theory. One can construct an analogue of Froyshov's correction term in the Pin(2)-equivariant setting; this is an integer-valued invariant of homology cobordism whose mod 2 reduction is the Rokhlin invariant.

- **I. Hambleton: Smooth group actions on 4-manifolds and Yang-Mills gauge theory**

  An equivariant version of the Yang-Mills moduli spaces can provide information and "hidden" constraints for smooth actions of finite groups on 4-manifolds. I will discuss this setting and present some sample results illustrating the difference between smooth and topological group actions.

- **W. Chen: Finite groups which act on S^4**

  In this talk, we will present some results on the following question: if a finite group G acts on S^4 continuously, is G necessarily isomorphic to a subgroup of $O(5)$? The talk is based on recent joint work with S. Kwasik and R. Schultz.

- **R. Gompf: Exotic smoothings of open 4-manifolds**

  Smoothing theory for open 4-manifolds seems to have stagnated in the past decade or two, perhaps due to the misperception that since everything probably has uncountably many smoothings, that must be the end of the story. However, most traditional approaches involve tinkering with the end of the manifold without probing the deeper structure such as minimal genera of homology classes. We show that this genus function, together with its counterpart at infinity, can be controlled surprisingly well compared to the case of closed 4-manifolds, and these tools are often complementary to traditional techniques.

- **O. Saeki: Broken Lefschetz fibrations and their moves**

  A broken Lefschetz fibration (BLF, for short) is a smooth map of a closed oriented 4-manifold onto a closed surface whose singularities consist of Lefschetz critical points (w.r.t. local complex coordinates compatible with the orientations) together with indefinite folds. Such a class of maps was first introduced by Auroux-Donaldson-Katzarkov in relation to near-symplectic structures, and now it is known that every closed oriented 4-manifold admits a BLF over the 2-sphere. In this talk, we give a set of explicit moves for BLFs, and give an elementary and constructive proof to the fact that any map into the 2-sphere is homotopic to a BLF whose indefinite fold has embedded image. This is joint work with R. Inanc Baykur.

- **D. Gay: Trisections of 4-manifolds**

  Trisections are to 4-manifolds as Heegaard splittings are to 3-manifolds. I will discuss existence and uniqueness, give examples, and raise lots of open questions. This is joint work with Rob Kirby.

- **R. Piergallini: Lefschetz fibrations over the disk**

  We provide a complete set of moves relating any two Lefschetz fibrations over the disc having as their total space the same four-dimensional 2-handlebody up to 2-equivalence. As a consequence, we also obtain moves relating diffeomorphic three-dimensional open books, providing a different approach to an analogous previous result by Harer. This is joint work with N. Apostolakis and D. Zuddas.

- **TJ Li: Virtual properties of 4-dimensional mapping tori**

  In this talk, we compute the virtual first Betti numbers of 4-dimensional mapping tori with prime fiber. As an application, we show that if such a manifold is symplectic with nonpositive Kodaira dimension, then the fiber itself is a sphere or torus bundle over circle. In a different direction, we prove that if the 3-dimensional fiber is virtually fibered then the 4-manifold is virtually symplectic unless its virtual first Betti number is 1. This is a joint work with Yi Ni.

- **D. Ruberman: Embeddings of non-orientable surfaces in a 3-manifold cross an interval**

  We use the correction terms from Heegaard Floer homology to obtain obstructions to embedding closed, non-orientable surfaces in 3- and 4-dimensional manifolds, focusing on embeddings carrying a non-trivial mod 2 homology class in a lens space or the product of a lens space and an interval. We show that the minimal genus for an essential non-orientable surface in the product of L(p,q) with an interval is the same as the minimal genus in L(p, q). This is joint work with Adam Levine and Saso Strle.

- **O. Plamanevskaya: Looking for flexibility in higher-dimensional contact manifolds**

  Contact structures in three dimensions play an important role in topology of 3- and 4-manifolds. By a classical result of Eliashberg, contact 3-manifolds come in two flavors, flexible ("overtwisted") and rigid ("tight"); the latter have an intricate relation to low-dimensional topology. In higher dimensions, a flexible class of contact manifolds is yet to be found. However, we show that manifolds containing a "plastikstufe" (aka "overtwisted family") exhibit certain flexibility properties. Joint with E. Murphy, K. Niederkruger, A. Stipsicz.

Junior Talks

	13:45-14:15	14:30-15:00	15:15-15:45	16:00-16:30
Wednesday	Anvari	László	Behrens	Lewark

- **N. Anvari: Equivariant gauge theory and the Poincaré homology 3-sphere**

  The Seifert homology 3-spheres bound canonical smooth definite 4-manifolds obtained by plumbing. We will show that certain free cyclic group actions on the Poincaré homology sphere do not extend smoothly, with isolated fixed points, to its even negative definite resolution. We use an equivariant version of the L^2-finite Yang-Mills moduli space for negative definite 4-manifolds with cylindrical ends. This is part of my PhD thesis, supervised by Prof. Ian Hambleton.

- **T. László: Poincare series, Ehrhart theory and Seiberg-Witten invariants associated with negative definite plumbed 3-manifolds**

  Let M be a rational homology sphere plumbed 3–manifold associated with a connected negative definite plumbing graph, hence, it is a link of a normal surface singularity. Motivated by analytic considerations, one associates with it a topological 'Poincare' series. This determines a special polytope as well. We develop the Ehrhart theory for it and discuss the relations with the Seiberg-Witten invariants of M.

- **S. Behrens: Smooth 4-manifolds and surface diagrams**

  Surface diagrams are a fairly new, combinatorial method to describe smooth 4-manifolds. They were discovered in the context of broken Lefschetz fibrations and, similar to Heegaard diagrams of 3-manifolds, they consist of closed, oriented surfaces together decorated with collections of simple closed curves. In this talk I will explain how surface diagrams encode topological information about 4-manifolds and indicate some interesting conclusions.

- **L. Lewark: Distinguishing the Khovanov-Rozansky knot concordance invariants**

  Replacing the sl_2 in Khovanov's link homology by sl_N for general N yields the Khovanov-Rozansky homologies. They induce knot concordance invariants for each N, the analogues of the well-known Rasmussen invariant. However, we will see knots for which they are not equal to the Rasmussen invariant, and give better bounds to the slice genus of a knot. The Khovanov-Rozansky concordance invariants detect the gap between the smooth and topological category. Geometrical applications of this property will be highlighted.